Learning from Africa: Mathematics

A report of Umalusi's research comparing Mathematics syllabuses and examinations in South Africa with those in Ghana, Kenya, and Zambia

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Contents

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Introduction

This short subject report is an addendum to the report emanating from an Umalusi study aimed at understanding how the South Africa senior secondary school certificate compares with those of three other African countries. The full research report is entitled, *Learning from Africa: A report of Umalusi's research comparing syllabuses and examinations in South Africa with those in Ghana, Kenya, and Zambia*. The research, in comparing South Africa's Matric certificate with the senior secondary school certificates of Ghana, Kenya, and Zambia, explored various aspects of the curriculum and examinations systems, including the intended and examined curriculum in four subjects.

The aim of the research was to learn from English-speaking African countries in different regions, in order to contribute to improving the intended and examined curricula in the Further Education and Training band in South Africa. Umalusi believes that it is valuable to understand our systems better by considering those in other countries, and hopes that this kind of comparative analysis will allow South Africans to stand back and achieve a distance from our internal debates. The research also cautions South Africa not to assume that our education system is superior to those found elsewhere in Africa.

The South African context of the research is a new curriculum which is in the process of being implemented in the Further Education and Training (FET) phase (senior secondary school). The FET phase, which covers the final three years of secondary schooling (Grades 10 to 12), culminates in the National Senior Certificate, the certificate which is to replace the current Senior Certificate. The implementation of the new curriculum began in Grade 10 in January 2006, and the first cohort of

Grade 12 learners will write the new National Senior Certificate in 2008.

The research aimed to understand how South Africa compares with the other countries, in terms of both the old the curriculum and examinations, which were still in use at the time of conducting the research, as well as the new curriculum. It attempted to understand what we can learn from the other countries with regard to systemic issues, as well as lessons for our new curricula and examinations on the basis of the subject comparison.

The study was conducted through meetings and open-ended interviews with officials in all four countries, supplemented by documentary information. Syllabus—and 2004 examination documentation was collected from each country and analyzed by groups of South African experts.

The full report provides a synthesis of what was learnt from the comparative study. It deals mainly with three issues:

- An overview of aspects of the education systems in the four countries. i.e. years in school, examinations and certification;
- A brief overview of comparisons of the intended and examined curriculum in four subjects at school-exit level, i.e. Biology, Science, English and Mathematics;
- Some reflections on the new curriculum in South Africa.

This short subject report, which provides a more detailed analysis of what evaluators found in their comparison of the Mathematics courses across the four countries, should ideally be read in conjunction with the main report.

The draft base report, which contains more detailed elaborations of the findings, is available on Umalusi's website as *Evaluating syllabuses and examinations: An Umalusi technical report comparing the syllabuses and examinations from Ghana, Kenya, South Africa and Zambia*, and may be of interest to subject experts.

Mathematics courses in the four countries

What type of Mathematics is appropriate at a secondary level, how much of it should be applied, how greater numbers of learners can be attracted to Mathematics and enabled to master Mathematics, and how much and what sort of Mathematics should be compulsory for all learners, are questions that have dogged policy-makers in South Africa and elsewhere for some time. This research offers some insights on some of these matters.

Table 1 provides an overview of the Mathematics courses offered in Ghana, Kenya, South Africa, and Zambia. These courses are not narrowly equivalent..

Table 1: Mathematics courses in the four countries

Certificates	Courses in the study	Mathematics courses not considered in the study	Status of Mathematics courses in certificates
Ghana Secondary Senior School Certificate	Mathematics (Core) Mathematics (Elective)		Compulsory Elective
Kenya Secondary School Certificate	Mathematics		Compulsory
Zambia School Certificate	Mathematics Additional Mathematics		Compulsory Elective
South Africa Senior Certificate (old)	Mathematics, Higher Grade Mathematics, Standard Grade		Both elective
South Africa National Senior Certificate (new)	Mathematics Mathematical Literacy	Additional Mathematics	Elective Compulsory, if Mathematics not been selected

Ghana and Zambia have compulsory Mathematics courses as well as an additional, more difficult optional course. Kenya has one compulsory Mathematics course. For the old Senior Certificate in South Africa there are two non-compulsory Mathematics course, Higher and Standard Grade Mathematics. There is also an Additional Mathematics curriculum which is offered at a limited number of schools, but whilst the contents and cognitive demands of this curriculum are considered more challenging than those in the Higher Grade syllabus, students are not required or even encouraged to take this course to gain admission to tertiary education courses. The elective Mathematics course from Ghana, and the Additional Mathematics course from Zambia, were included in the analysis, but the Additional Mathematics course from South Africa was not. This is because the Ghanaian and Zambian courses have more in common with South African Mathematics courses, given the fact that the other Mathematics course on offer in these three countries is compulsory.

The Mathematics courses in Ghana, Zambia, and South Africa span the three years of senior secondary school, while the Kenyan course is taught over four years. These differences, and other variables, made a direct comparison of the courses a very complex activity. Nonetheless, the research did raise some interesting points for policy-makers.

Overview and discussion

The differences in the optional/compulsory status of courses means that the numbers of learners writing Mathematics varies dramatically across the countries, and, perhaps more significantly, the percentages of the cohort taking Mathematics varies.

The populations of the three countries are roughly in the following ratio:

South Africa: Ghana: Kenya: Zambia: 1 000: 472: 732: 242

If Mathematics were compulsory in all countries, one might expect the ratio of candidates writing some Mathematics at senior secondary level to be comparable. The actual ratio of the number of candidates taking Mathematics (at one or more grades or levels) is:

South Africa: Ghana: Kenya: Zambia: 1 000: 350: 795: 318

What is surprising, perhaps, is that despite being a compulsory component, the ratio of Ghanaian students who complete a Secondary Senior Certificate with at least core Mathematics is much lower compared to the other countries (even lower than South Africa where Mathematics is not compulsory). The ratios of students in Kenya and Zambia completing a Mathematics course at Senior Secondary level are higher compared to the total population than is the case in South Africa, which is to be expected, taking into account that only just over 50% of South African Senior Certificate candidates offer Mathematics as one of their subjects. Despite Mathematics not being compulsory, there is a high failure rate in South Africa, which is of great concern.

The Kenyan, Ghanaian, and Zambian Mathematics courses which are compulsory for all learners are broader than the various South African Mathematics courses. However, in general, the core curriculum in each of the other countries appears to be tested in less depth than is the case for the old South African syllabus. This makes sense for compulsory Mathematics courses. In the new curriculum in South Africa, we have gone a different route: a specialized Mathematics course which is optional, and a Mathematical Literacy course that aims to provide broad mathematical skills to all learners.

Whether South Africa has chosen the right approach in going the 'mathematical literacy' route surfaced in this research. One of the evaluators argued that the Kenyan syllabus, with its strong arithmetical focus, would be more appropriate for South Africa. Other evaluators believe that the emphasis on contextual problems in the new Mathematical Literacy curriculum is really important. They maintain that there is an arithmetical focus, though different from the Kenyan one. There is a concern, however, that reading and comprehension in a second language could make the contextual problems too difficult for those who do not have the benefit of studying in their home language. Evaluators argued that the success of the Mathematical Literacy curriculum will be determined by the exemplars and examination papers produced by the examination panel. These will provide critical guidelines for teachers. They also felt that needs to be a good proportion of questions focused on basic arithmetic which require little or no language comprehension.

The intended curriculum

Aims/purpose/vision/outcomes

While the Mathematics courses tend to have far-reaching and broad aims, there is often little in the actual body of the syllabus that enables teachers to achieve these aims. In other words, syllabus aims seem to be an 'add-on', and syllabuses are not designed to enable teachers, in fact, to achieve the aims. It could be argued that this is not required in a listing of content and that teachers need to attend to the aims in the way that they teach.

The aims of the various Mathematics courses range from four sentences to two and a half pages. Six main aims, common to all (or most) of the countries were identified. Significantly, all curricula claim to prepare students for further education and/or work. Another common aim is speed, accuracy and skill in performing mathematical calculations and manipulations. All courses also claim to teach abstract thinking. All courses aim to develop in learners the ability to investigate, generalize, analyse, reason, make conjectures, prove, and make predictions. All courses aim to build competences in mathematical modelling or applied Mathematics or every day life applications. Finally, all the courses except the Ghanaian ones aim to teach communication: the South African course refers to the ability to 'understand, interpret, speak and write mathematical language'.

Noble aims are stated in the new South African curriculum statement, and it is hoped these will be realized in practice. The vision that is projected of Mathematics is to make available to the learner "extended opportunities" that would make the learner's achievements comparable to international standards.

The Mathematics evaluators argued that, according to their experiences in South Africa, teachers pay little attention to the substantive policyorientated aims of the curriculum if the syllabus does not explicitly contain ways of achieving these aims in the body of the curriculum (and not just mentioned as part of an introduction). Furthermore, teachers "teach to the test" and so the onus lies to a large extent, on examiners and subject advisers to make sure that examinations and/or externally set curriculum tasks address the stated aims of the curriculum, although to some extent this is obviously impossible—for example, Kenya includes amongst its objectives the need to "develop a willingness to work collaboratively", something which obviously cannot be tested in an examination. In South Africa, attempts are being made to include group work in portfolios, but it is hard for teachers to determine what the contribution of each individual was to what the group discovered and not just what the final written submission was worth, and it is very unclear how reliable and valid the marking is from many South African schools.

Content coverage

A summary of the content covered in the various courses and grades in the various countries can be found in the detailed technical report of this project, available at www.umalusi.org.za. Topics which are in the basic courses in all the other countries, but are not in the South African curriculum are:

- The use of logarithmic tables
- Matrices
- Statistics and probability
- Transformation geometry
- Vectors.

Of these topics, only statistics and probability have been included in the new curriculum statement for Mathematics. The use of logarithmic tables has been replaced by the use of scientific calculators and it was felt that the other topics receive only superficial treatment in other countries and will best be left to tertiary courses.

Topics that were in the old South African curriculum and not in most other countries include:

- Quadratic inequalities
- The theory of logarithms
- Inverse functions
- Absolute value
- More complex trigonometric graphs
- Three dimensional trigonometric problems
- Compound angles.

The theory of logarithms and absolute value have been excluded in the new National *Curriculum Statement* to make space for other topics which were considered more useful to real life applications.

One evaluator suggested that South Africa might consider following the Kenyan example when it comes to geometry. The current difference is that the Kenyan approach is practical whilst the South African syllabus emphasises riders which lean towards the abstract. Given the abysmal pass rate in Paper 2 in South Africa, the researcher suggests that curriculum designers might shift the emphasis from the abstract to the practical aspects of geometry, including scale drawing.

In the new South African Mathematics *Curriculum Statement*, topics which have for some time been included in the curricula of other countries have been introduced for the first time in the senior grades: transformation geometry, and statistics and probability are examples. There is also an attempt to transform the way Mathematics has been taught from an approach which was very content-related and traditional, involving mostly the direct teaching of isolated bits of knowledge, to an approach that is learner-centred and integrated. Evaluators felt this was a good move. Learners' backgrounds and the contexts of their schools are emphasized as well as the inclusion of indigenous knowledge, human rights and issues of social justice and the transformation of society. Some topics like the nature of roots, absolute value, and complicated algebraic manipulation for its own sake have been excluded to make room for the newer topics.

All countries except South Africa include Statistics and Probability in their core curricula. This omission has been addressed in the new South African National Curriculum Statement. This topic has been included in both the Mathematics and the Mathematical Literacy curricula. They are currently not part of the compulsory section of the curricula and will be examined only in an optional question paper until about 2010. It is understood that this is to give teachers time to develop their own competence in these content areas.

The challenge for the designers of the Mathematical Literacy curriculum was greater. Deciding what Mathematics should be included to serve the stated purpose: to "…engage with real life problems in different contexts, and so to consolidate and extend their basic mathematical skills" (*Mathematical Literacy Curriculum Statement: 2003*) must have been very difficult. It was hoped that the subject would serve learners who have shown themselves to be competent in Mathematics in the lower grades, but don't intend following a scientific career path, as well as those who had great difficulty with the subject. A significant emphasis has been placed on real world applications on Mathematics. Nonetheless, there is still a considerable amount of Mathematics which must be mastered and applied by learners. One evaluator argued that *if* Mathematical Literacy is adequate for learners who wish to take Physical Science as one of their subjects, the trigonometry, which has been excluded for the first few years needs to be re-included as soon as possible. Trigonometry is also needed for learners wishing to take certain courses (like Architecture) at tertiary institutions.

In both new curriculum statements, there is content which is in the curriculum but will not be tested in the initial years of implementation. A 'core' set of topics is examinable immediately. This pause is to ensure enough time for teachers to master all aspects of the new curricula. While this is a pragmatic decision on behalf of the Department of Education, it will need to be carefully managed if confusion is to be avoided.

It seems from the topics that were included in the 2003 *Mathematical Literacy Curriculum Statement*, but have subsequently been declared not part of the core curriculum for the first few years, that some reviewers thought the curriculum overloaded with topics, or that some topics would be too demanding or would be beyond the initial capacity of the teachers who would be required to teach the subject. Evaluators argued that the number of Senior Certificate candidates who, in recent years, have not taken

Mathematics at any grade, in the last three years of secondary schooling, would indicate that there are not enough qualified Mathematics teachers to teach all those who would previously have dropped Mathematics.

Evaluators argued that it is difficult to make judgements about cognitive complexity within syllabuses in the absence of examinations. Even though certain topics are inherently more complex, the inclusion of topics in a syllabus is misleading, as they could be examined in a very superficial manner.

Coherence

None of the Mathematics courses appeared to have any overall principle holding them together, but Mathematics curricula worldwide include:

- numerical work
- functions and algebra
- geometry and trigonometry
- data handling (ie statistics and probability).

The old South African curriculum, which will have been phased out by 2008, was strong in the first three categories, but, as has already been mentioned, data handling was done only in junior grades, and more often than not, was omitted altogether as it was not examined at Grade 12 level.

It was not always possible to judge from the curriculum documents themselves why topics have been sequenced as they have, what principles underpin the inclusion and exclusion of certain topics from the curriculum and how the aims or general objectives can be achieved through the teaching of the various topics. The mathematical content rather than processes are foregrounded in all the documents studied. Evaluators felt that more guidance needs to be given about mathematical processes.

The Zambian compulsory Mathematics curriculum document indicates that Mathematics is not seen as a subject separated from society and societal issues, particularly those that are pertinent to matters of '…national concern such as Environmental Education, Gender and Equity, Health Education and HIV/AIDS…' (Curriculum Development Centre, 2000 p iii), but the apparent link between Mathematics and the specified societal issues raises a critical question about *how* the content of Mathematics and its assessment processes actually address the purported connections. Similar claims are made in South African documents: '…work towards the reconstruction and development of South African society…create an awareness of and a responsibility for the protection of the total environment…' (1995 Interim Core Syllabus), but there is no indication in the body of the syllabus as to how these aims might be achieved. This issue might be handled in other documents that were not provided.

Sequencing, progression, and pacing

Sequence and progression appears to be very similar in Mathematics courses. But the order in which topics are listed in a syllabus for a particular grade does not necessarily have particular significance. Some teachers devote one or more periods every week to each of the strands: algebra, geometry etc, whereas others complete a topic from algebra, say, before progressing to a different topic in geometry or trigonometry. Very few teachers follow the curriculum for a grade in the order in which it is set out, completing, say, all the algebra before starting with some geometry.

Only the Kenyan and Zambian documents indicate the number of teaching periods to be devoted to each topic. In each case the time devoted to arithmetic and number decreases (though not uniformly in the case of Kenya) over the four/three years of secondary education and no time is devoted to this topic in the final year. It is presumably assumed that students have mastered the concepts and can use them in a variety of situations. In both cases, the time spent on geometry in the final year is significant. Although the actual periods devoted to each topic are not available for Ghana and South Africa, evaluators argued on the basis of their experience that the trends are similar in South Africa. However, they felt that in the case of geometry, the strong emphasis on developing a logical argument in Euclidean geometry makes this component of the South African curricula more cognitively demanding than appears to be the case in any of the other countries.

For the old syllabus in South Africa, documents supplementary to the syllabus, developed by provincial departments of education, are there to assist teachers with regard to pacing within and across years. It is likely that such support is also provided in Ghana. It was interesting to note that there is a considerable difference in the total number of teaching units prescribed across the four years in Kenya, for example: 215 in Form 1, 242 in Form 2, 273 in Form 3 and 204 in Form 4. Evaluators speculated that in the forms with fewer periods allocated to specific topics, there may be more time for revision and activities that develop the learner's ability to tackle mathematical investigations, to enable them to make conjectures and try to justify or prove them.

Evaluators felt that the new curriculum statements in South Africa had a clear sense of cohesion between and within grades. Information about pacing is included in the *Learning Programme Guidelines*.

Assessment specifications

In Ghana, a two-dimensional grid is supplied for examiners, with rows for content coverage and columns for four types of thinking, labelled A, B, C, and D. The document gives no indication of what these categories are, or in what ratios the content and cognitive categories need to be covered. The ratio of the examination to the year mark is 7:3, but there is no information in the documents provided as to how the year mark is compiled.

Extensive notes have been provided with the Kenyan syllabus. Teaching approaches and resources are suggested and time allocation is given as well as indications of assessment. It is noted that the assessments suggested are almost exclusively time-constrained written tests, with no suggestion of any open-ended tasks to be investigated or projects involving the collection and analysis of statistical data. A Table of Specification requires examiners to set questions which satisfy the requirements of a two-dimensional grid, with each cell indicating the number of questions (0, 1 or 2) to be set in each content area, with a specified cognitive demand: Knowledge, Comprehension, Applications, Analysis, Synthesis or Evaluation. Each row and each column is also required to satisfy a given percentage. Whilst the overall distribution of content and cognitive demands are more or less achievable, evaluators felt that it would be almost impossible to identify questions in Mathematics which satisfy the severe constraints of the grid.

No specifications appear in the documents available for Zambia as to the ratio in which content topics should examined nor of the ratio in which questions should be set at different cognitive levels.

In the old South African syllabus, content percentages are prescribed, which allow for minimal variation in some areas, but in other areas the percentages are fixed. The *National Guidelines* (2002) identify three cognitive categories: 1: Knowledge and Skills, 2: Understanding and 3: Application and Creative Thought. Standard Grade papers need to cover these cognitive categories in the ratio 50:40:10 and Higher Grade papers in the ratio 40:40:20. The ratio of the examination mark to the mark for a portfolio of work done in Grades 11 and 12 in South Africa is in the ratio 3:1. There are guidelines indicating how the portfolio needs to be compiled. In addition to the traditional tests and examinations, the portfolio must include investigations, group work and a project. Some work must be done under supervision and some can be done at home.

The new South African Curriculum Statements provide detailed specifications for assessment. Three forms of assessment are listed in the guidelines. These are: observation-based, performance-based, and test-based assessments. It is noted that

> observation-based assessment requires that learner performance be assessed while the learner is actually performing a skill in the classroom as there will be no concrete product for the teacher to assess after the performance (p. 17).

Evaluators questioned how can this "observation-based" assessment can be realized in practice, and what methods were available to the teacher to achieve this critical form of assessment given that it is both formative and highly contested though obviously interesting and can capture learning uptake as it happens.

Evaluators felt that the *two* examinations proposed in Grade 12 (over and above the final externally set and marked examination) is thought by most to be a waste of valuable teaching time. One examination, some time between June and September has proved perfectly adequate in the past.

Evaluators argued that the nature of Mathematics is such that it is difficult to make it clear what degree of complexity in calculations should be demanded. The intention of the designers of the new curriculum was that questions involving excessively complicated, time-consuming manipulation should be avoided, to make time for new topics and for the development of big ideas to replace the acquisition of fragmented pieces of knowledge. For example in the Mathematics Higher Grade first paper of 2001, candidates were required to simplify:

$$\left(\frac{\log x}{\log 2}\right) = \log_2 \left(\frac{8}{x^2}\right)$$

Several manipulative skills are tested here, but the question is contrived: it would not arise in a contextual framework and seeks to identify whether the candidate has identified the different manipulations permissible within the laws of logarithms. The move away from very complex manipulation for its own sake is not explicitly stated in the *National Curriculum Statement* and teachers are reluctant to omit the kind of questions which have featured in examinations based on the curriculum which is being replaced. They are generally guided by the textbook that they choose or which is supplied by the Department. Texts vary dramatically in approach, cognitive demands, complexity of manipulations and mathematical correctness. This might be interpreted as showing that the *National Curriculum Statement* document, in particular, has insufficient detail relating to content, but textbook reviewers seem to have been more concerned with technical detail and evidence of transformational philosophies than with actual content. One evaluator argued that books which handle many topics very superficially or which actually omit sections of the curriculum, have been approved. For example, text A devotes 20 pages to probability in Grade 12 which include 12 exercises or activities ; text B devotes 10 pages to the topic which contain 9 activities or exercises and text C has excluded the section altogether.

General user friendliness of documents

Evaluators did not find problems with the syllabus documents from Ghana, Kenya, and Zambia, or the old South African syllabus, but commented on some problems with the new South African curriculum statements. The number and extent of the documents is quite daunting and there is a good deal of repetition and some contradictory statements that will be confusing for teachers. For example, chapter 4 in the *National Curriculum Statement* seems to have been superseded by the *Assessment Guidelines*. The former document refers to the *Qualifications and Assessment Policy Framework for Grades 10 – 12 (General)*. The researchers could not obtain access to this document, and did not understand how it fits in with the other documents. Chapter 4 of the *National Curriculum Statement* also refers to *White Paper 6 on Special Needs Education: Building an Inclusive Education and Training System*, which means that teachers of learners with special needs need to access yet another document.

Evaluators noted some contradictory or confusing statements. For example, the *National Protocol for Assessment*, published in December 2006, reflects the need, in Grade 12, for examinations in the second and/or third terms, and the Mathematics and Mathematical Literacy *Subject Assessment Guidelines*, published in January 2007, omit the "/or". Some schools, subject advisors and other provincial officials have erroneously used this document to support their claim that examinations **must** be written in three of the four terms in Grade 12. The Department of Education points out that the former document (which is gazetted) is the one to be followed which means that schools can choose whether to write one or two sets of internally set examinations before the final external examination in the fourth term, but not all teachers or subject advisors know this. Losing teaching time to schedule so many examinations will certainly impact on the quality of teaching and learning. However, most teachers, evaluators argued, understand that the *National Curriculum Statements* and the *National Protocol for Assessment* are policy documents, and that the other documents are guidelines.

Evaluators felt that many of the problems being experienced at present by users of the Mathematics documents are caused by the fact that there are several versions of both the *Learning Programme Guidelines* and the *Subject Assessment Guidelines* in use. The Department needs to make sure that teachers are working with the latest document (at the time of writing the latter two documents, for both Mathematics and Mathematical Literacy are dated January 2007). Modifications have been made by various officials and these have important implications for teachers, examiners and subject advisors. A teacher working just from the *National Curriculum Statement* would not know that several topics have been identified as optional, until further notice. Some are concerned that the optional topics will never be made compulsory and that if this happens, the curriculum will not be balanced (nor will the curriculum be comparable to modern Mathematics curricula internationally). One evaluator pointed out that it was mentioned at a recent conference of the South African Mathematics Society that judging from the Grade 12 exemplars of the Mathematics papers, paper 1 is similar in its mathematical demands to the 'old system' paper 1 but paper 2 appeared very routine and was far less challenging than the current paper 2. This would not be so if sections which are currently optional (such as more advanced statistics, probability and Euclidean geometry) become compulsory.

The January 2007 of the Mathematics *Learning Programme Guidelines* includes excerpts from possible work schedules for Grades 10, 11 and 12, as well as examples of full work schedules for the three grades. It is not clear what the point is of having excerpts as well as full schedules. Perhaps the intention is to show slightly different styles of work schedules, where one includes "lesson focus and activities", whilst the other has a column that can be filled in indicating resources to be used and another column in which it is suggested that dates and or the nature of the assessment task that will be used. In addition, earlier versions of the *Learning Programme Guidelines* for Mathematics included exemplars. But these are no longer part of the document. There is, however, a fairly detailed *Teacher's Guide for Mathematical Literacy* which gives examples of tasks that could be set.

Evaluators argued that it is not clear on what criteria assessment standards were allocated to the core or optional groups. There is a statement in the Mathematics assessment guidelines (September 2005) that the motivation for not implementing the full curriculum immediately was to give teachers time 'to develop their capacity'. Assessment standards 10.3.2, 11.3.2 and 12.3.2 refer to topics in Euclidean geometry that are in the old curriculum and are therefore not new to teachers. The implication seems to be, then, that teachers need to improve their capacity to teach this aspect of learning outcome 3 more effectively. Is this to be the responsibility of the teacher, at his/her own expense, or will the Department of Education or provincial or district officials assume this responsibility? How will teachers gauge their own capacity? By the performance of their learners? Perhaps some form of evaluation of teachers' capacity needs to be implemented to avoid a 'one size fits all' professional development strategy?

Evaluators also argued that it is only once a curriculum is in use that ambiguities and other difficulties are exposed: for example, points of inflection are to be considered as mentioned in parenthesis, in relation to stationary points on the graphs of functions. The intention of the curriculum is that points of inflection be considered in their own right so as to develop a proper relational understanding of points of inflection as points at which concavity changes. Some are arguing that only horizontal points of inflection need to be taught, because oblique points of inflection do not occur at stationary points. This is true if a stationary point is defined to be a point where function values are neither increasing nor decreasing and the first derivative is zero. But it could be argued that derivatives are also functions and the stationary point/s of the gradient function or first derivative need to be considered in relation to change in concavity. If points of inflection are originally taught only in relation to those which occur with horizontal tangents, misconceptions arise which are very difficult to remove. The big idea of inflection then gets lost.

Evaluators also pointed out that the *Curriculum Statements* for Mathematics and Mathematical Literacy are 91 and 72 pages long respectively, the *Assessment Guidelines* 33 and 31 pages, and the *Learning Programme Guidelines* 29 and 33 pages. *The National Protocol* is 27 pages, and the *Teacher's Guide* for mathematical literacy is 69 pages. The length of the documents, together with the other problems pointed out above, suggest that the curriculum documents in the other African countries are simpler and easier for teachers to use is true. However, evaluators argued that in the long term, the new curriculum will be a vast improvement on the old one, and were very positive about its ability to develop cognitive mathematical skills such as investigating, generalising, making conjectures, proof, modelling.

The examined curriculum

General

Which?? The compulsory and elective courses are both tested through a multiple choice test of 1,5 hours and a second paper of 2,5 hours, requiring fully worked solutions. All major mathematical topics have been covered, although one of the evaluators argued that trigonometry is not adequately covered in the compulsory course. The compulsory syllabus states that "sine and cosine rules may be used" but exactly what is required is not clear. Some advanced topics are included in the elective course. These include binary operations, kinematics and integration.

In Kenya, the single Mathematics course is tested through two papers, each bearing 100 marks. These papers cover four years of work. Although the curriculum statement indicates that Paper 1 "covers mainly Form 1 & 2", our analysis reveals that about 33% of Paper 1 covers work done in Forms 3 & 4.

It was noted that electronic calculators are not allowed in either of the Kenyan papers (this has subsequently changed, as the curriculum in Kenya has been revised since the writing of this report). At least one of the researchers believes that the numeracy skills of Kenyan learners would have been enhanced through this 'no calculator' policy, arguing that the experience of tertiary Mathematics educators in South Africa is that students are overwhelmingly incompetent in performing even trivial calculations and this can be attributed to the unrestrained use of calculators. Three of the evaluators disagreed with this, and favoured the South African policy. The dissenting voice, however, felt that that learners' numeracy skills are enhanced by forbidding calculators, and felt that South Africa urgently needed to improve learners' ability to perform calculations.

Mathematical modelling and the development of strategies and tools to solve problems are, evaluators argued, the two highest principles of the discipline. An analysis of the two 2004 Kenyan papers reveals that much emphasis has been placed on problem-solving.

Zambia has two Mathematics courses: one is compulsory and is referred to as "Syllabus D" on examination papers and the second is an optional Additional Mathematics course. Both courses are examined by a 2-hour paper worth 80 marks and a 2,5 hour paper worth 100 marks. No calculators are permitted in Paper 1 of the compulsory course.

Under the intended curriculum discussion above, the argument of one of the evaluators in favour of including more Arithmetic in Mathematics was made. The evaluator in question argued further that the lowest cognitive level named "remember" includes memorization of simple addition and multiplication tables, which are crucial in all calculations. It includes the trigonometric ratios of all the principle angles and a host of other little facts that are fundamental to progress in Mathematics. Learners see no need to remember these tables and facts as calculators provide quick answers. The result is that they struggle within complex procedural tasks, like multiplication and division of polynomials, simplifying algebraic expressions, working with ratios, and doing simple calculations in commercial Mathematics involving percentages etc.

Evaluators further argued that whilst multiple choice tests are easy to mark, often the skill which is being tested is not the skill being used by the candidate. Often substitution of possible answers produces the correct answer more quickly and easily than applying the intended procedure. For example one question (Ghana Elective 2001: question 7) reads as follows:

> R$(-3,1)$ and S$(-1,3)$ are two points on a circle. If RS is a diameter, find the equation of the circle.

The examiner almost certainly expected the candidates to:

> determine the coordinates of the midpoint (x_M, y_M) of the diameter, the length of RS $= \sqrt{(-3+1)^2+(1-3)^2}$ and hence the radius of the circle and then to form the equation using:

the formula $(x-x_M)^2+(y-y_M)^2 = \text{radius}^2$, multiply out and find the corresponding equation amongst the four options. But the coordinates of R satisfy only the correct equation, which saves all the above work! Some weaker candidates become very good at this. Of course very sophisticated test development could avoid this type of problem, but in the South African experience this has not been easy to achieve—possibly why objective test items are no longer used in South African Senior Secondary Mathematics examinations.

Comparison of the four countries reveals that the cognitive demands of the Kenyan and South African Higher Grade papers are closest with Kenya having 27% of the questions categorized as Problem-Solving, compared to South Africa Higher Grade 18%, Ghana compulsory course 14%, Ghana elective course 13%, Zambia Additional 11%, South Africa Standard Grade 9% and Zambia Syllabus D only 4%. Combining the two more demanding categories results in the following ranking: South Africa Higher Grade 48%, Kenya 47%, Zambian Additional 47%, Ghana Elective 45%, South Africa Standard Grade 32%, and Ghana compulsory course and Zambian D syllabus only 22%[1].

The highest cognitive demands in terms of the adaptation of Bloom's taxonomy are found in questions which require candidates to analyze, evaluate and create. The percentages in this category, in order, were: South Africa Higher Grade 21%, followed by Kenya 13%, South Africa Standard Grade 11%, Ghana Elective 8%, Zambia Additional 7%, and Ghana compulsory and Zambia Syllabus D 4%. The Zambian Syllabus D included the highest percentage (49%) of questions that involved remembering. The Ghanaian compulsory examination had 38% in this category and Kenya 30%. The South African papers had a lower percentage in this category: 22% in the Standard Grade paper and 23% in the Higher Grade paper, but it needs to be remembered that, whereas all candidates in the other three countries write examinations on at least one of the Mathematics courses, this was not true in South Africa at the time of the

[1] See Umalusi (2007) *Making Educational Judgements: Reflections on judging the standards of intended and examined curricula* for a detailed discussion of the ways in which evaluators categorized levels of difficulty and cognitive operations.

research. Only from 2008 will all candidates writing the exit examinations at the end of Grade 12 be required to include either Mathematics or Mathematical Literacy in their subject selections.

As would be expected from the fact that Mathematics in South Africa is optional, while it is compulsory in the other countries, some of the main topics in the South African curricula appear to be handled much more superficially in the Ghanaian, Kenyan, and Zambian examinations. The other countries in fact include a wider range of topics but none is covered in the same depth as geometry, algebra, calculus and trigonometry are in the South African curriculum, although the Ghanaian Elective and Zambian Additional Mathematic courses ascribe significant importance to calculus (both differential and integral). In support of this claim, questions on Euclidean geometry, the solution of triangles in trigonometry and calculus from the four countries were compared. On the basis of this analysis, evaluators argued that there is no doubt that the South African question is significantly more demanding than the three from the other countries. The Kenyan approach to riders in geometry seems to have a lot to offer South African curriculum designers, because of the practical emphasis.

In the case of the trigonometry questions analyzed, evaluators found that the Ghanaian and the Zambian questions are very straightforward two-dimensional problems where the sketch has been drawn and the relevant formulae can be applied directly. The Kenyan question is more demanding as it is three-dimensional and although the calculations are simple, they require the identification of the correct triangles, which are not drawn in. All triangles are right-angled and consequently the sine and cosine rules are not required. The South African question is also a three-dimensional problem and although the triangles involved are not all right-angled and the sine or cosine rule must be used, it is not significantly more demanding than the Kenyan one. The range of trigonometry questions in the South African paper is, however, much broader and includes equations, identities, compound angles and graphs. The Kenyan paper covers a much wider range of other topics.

With regard to calculus, although only differentiation is prescribed in South Africa, this is handled in more depth than in the other countries where the emphasis seems to be on the techniques of differentiation and/or integration, rather than on the underlying concepts like limits and differentiation from first principles. Evaluators claimed that although the other three countries include integration in their courses, this is at a very basic level. Differentiation seems to be limited to the solution of typical problems on maximization and rates of change. The Zambian questions are more demanding than those of the other two countries and overall might be more demanding than the South African questions, but they are easier than the calculus questions in South African Additional Mathematics papers. What follows is a brief comparative discussion of examples from the four countries:

- Ghana 2001 Elective Paper 2, question 12: The question requires the application of the rules for differentiation and integration. The functions, once simplified are polynomials and are therefore easy to differentiate and integrate.
- Kenya Paper 1, question 5 and Paper 2, questions 13 and 22: Both questions are distance, velocity and acceleration problems and question 13 asks for the equation of a tangent, given the coordinates of the point of contact and the gradient function.
- South Africa Higher Grade Paper 1, questions 6 and 7: None of the questions is particularly demanding, but there is a range of questions, including differentiation "from first principles", some fairly straightforward functions to differentiate (not all polynomials), a cubic graph to interpret, a question on the rate of change of the depth of water in a dam and a problem which required the calculation of the dimensions of the rectangle of maximum area that can be placed between the x-axis and a given parabola. Although the last mentioned problem is scaffolded by questions requiring 'm to be expressed in terms of x' and 'show that the area is …', more is required than the application of formulae in very familiar types of problems.
- Zambia Additional Mathematics Paper 2, questions 8, 9 and 10: The three questions include the calculation of the area between a straight line and a parabola, the volume of the solid formed by

rotation of this area about the x-axis, the maximum volume of a rectangular box where the surface area was given and the length and breadth of the base were given as $2x$ and $3x$. Scaffolding was provided, similar to that in the South African problem in that candidates were asked to show that the height was…. The first part of the third question was a standard distance, velocity, acceleration problem and the second part was a common type of rate of change question.

In general, it was felt that South Africa's examinations stand up well in comparison to the other countries, in terms of types of knowledge and cognitive operations tested, as well as in terms of levels of depth and difficulty. However, it must be borne in mind that both the Higher and Standard Grade courses are optional, whereas all other countries have Mathematics courses that all learners write.

Further, finding an appropriate balance between depth and breath, abstract and applied, and between basic skills and highly challenging skills, are going to be ongoing challenges for South African Mathematics curriculum designers.